Writing a System of Linear Inequalities

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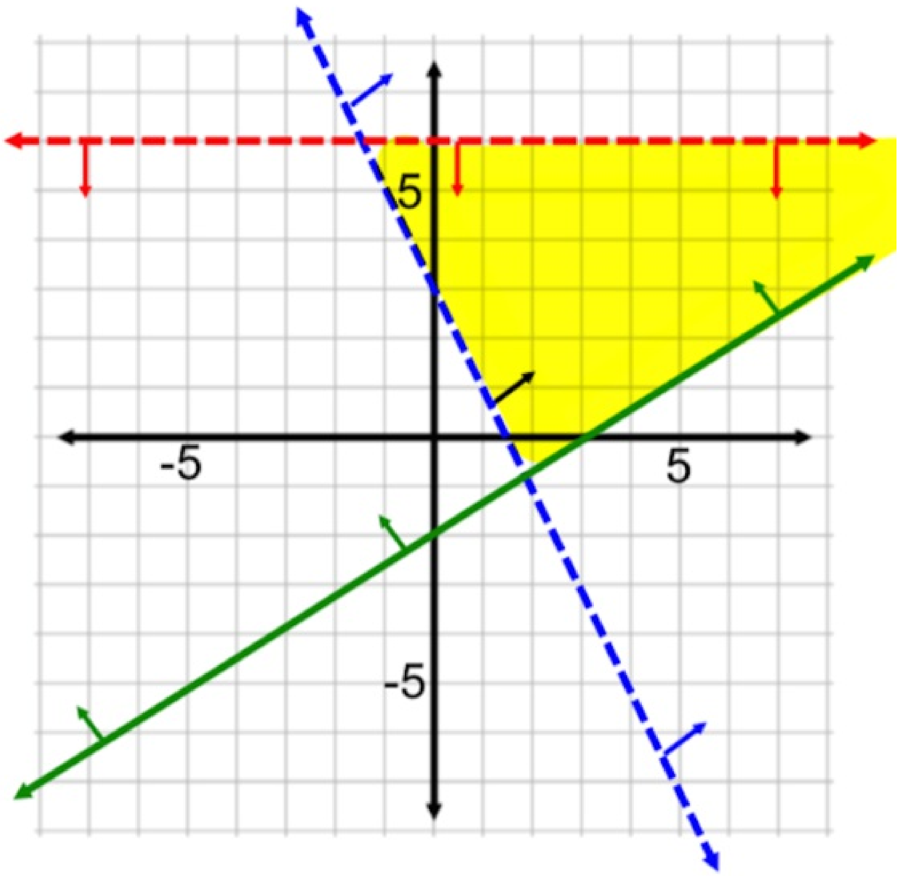
WHAT'S COVERED

In this lesson, you will learn how to determine the system of linear inequalities that represent a given situation. Specifically, this lesson will cover:

1. Writing a System of Linear Inequalities from a Graph

In order to write the inequalities that make up this system, we need to look at each individual piece of the graph and construct an inequality.

EXAMPLE Write the system of inequalities from the graph below:



Let's focus on the blue inequality first. The first thing to note is that the boundary line is a dashed line. This means that the inequality symbol will be either < or > as it is strict and does not include the exact points on the boundary line. We can also note that the shaded region is above the blue line, which indicates that all coordinate points above the boundary line satisfy the inequality. This means that the inequality symbol is >, or the "greater than" symbol.

Now, we can think of the boundary line as an equation, write an equation in the form $y = mx + b$, and simply replace the equals sign with our inequality symbol. To write the equation in the form $y = mx + b$, we need to find the slope and the y-intercept.

To find the y-intercept, we look at where the line crosses the y-axis. We see that it does so at the point (0, 3), which means our b-value in the equation is 3. Next, we calculate the slope by counting rise and run to get from one point to the next. Let's start at our y-intercept, and count the rise and run to get from the y-intercept to a discernible point on our graph, say (1, 1). The rise is -2 and the run is 1, making our slope $\frac{-2}{1}$, or 2. The equation of the boundary line is $y = -2x + 3$. Now replace the equal sign with our inequality sign > and we have determined one inequality in the system:

$$y > -2x + 3$$

We follow the same process with the other boundary lines. Let's work with the green line next. We notice a y-intercept of -2. To find the slope, let's count the rise and run from the y-intercept at (0, -2) to another point on the line, say (3, 0). This is a rise of 2 and a run of 3, making our slope $\frac{2}{3}$. This creates the equation $y = \frac{2}{3}x - 2$. Our boundary line is a solid line, narrowing our inequality symbol to either ≤ or ≥. Since the solution region to the system includes all points above this line, we know the inequality symbol is ≥. Replace the equal sign with our inequality sign ≥ and we have determined another inequality in the system:

$$y \geq \frac{2}{3}x - 2$$

Finally, let's focus on the red boundary line. This line is a horizontal line, so there is actually no x-term in its equation. The boundary line is simply $y = 6$. To turn this into an inequality, we note the dashed line, which indicates that it will either be the symbol < or >. Since the solution region is underneath the line, we will use the <, or the "less than" symbol. Replace the equal sign with our inequality sign < and we have determined the third inequality in the system:

$$y \leq 6$$

Putting all three inequalities together, we have the following system of linear inequalities:

$$\begin{aligned} &y > -2x + 3 \\ &y \geq \frac{2}{3}x - 2 \\ &y < 6 \end{aligned}$$

2. Writing a System of Linear Inequalities from a Scenario

We can create a system of linear inequalities to express real-world situations.

↱ EXAMPLE You manage an art store and need to order canvas paper and brushes for the studio. You must buy at least 200 sets of paint brushes, which cost $15 each, but no more than 75 rolls of canvas paper, which cost $30 each. Your budget for this expense is $5,000.

We can represent these budget and quantity restrictions with a system of inequalities. The first thing we need to do is define variables. We are purchasing sets of paint brushes and rolls of canvas paper, so we will make these the definitions for *x* and *y*:

$x = $ sets of paint brushes
$y = $ rolls of canvas paper

One of our limitations in our scenario is our budget. We cannot spend more than $5,000 on these supplies. This means that the total cost of supplies must be less than or equal to $5,000 (because our bill can be exactly $5,000 and still fit within our budget). To write this inequality, think about how we would express the total cost for paint brush sets and canvas paper rolls using the above information, while also restricting it to less than or equal to $5,000. We can express this as:

$$15x + 30y \leq 5000$$

Note that to express cost, we multiplied *x* and *y* by their respective costs: $15 for each paint brush set, and $30 for each roll of canvas paper.

We also have additional restrictions from our scenario. These restrictions involve quantity: we must buy at least 200 sets of brushes and no more than 75 rolls of canvas paper. We'll set up a relation between *x* and 200, as well as *y* and 75.

Let's look at the first part of the above statement: we must buy at least 200 sets of brushes. "At least" means that we can include the exact value of the limitation and anything above. In other words, this means that buying 200 paint brush sets is okay. Even buying 300 sets of paint brushes meets this restriction (although it does put our other restrictions at risk). Thus, our second inequality in the system is

$$x \geq 200$$

Now let's revisit the second part of that statement: we must buy no more than 75 rolls of canvas paper. "No more than" also means that we can include the exact value of our limitation any anything below. In other words, this means that we are allowed to buy up to 75 rolls of canvas paper, but not 76, or anything higher. So the final inequality in the system is:

$y \le 75$

Our system of inequalities for our situation is:

$$15x + 30y \le 5000$$
$$x \ge 200$$
$$y \le 75$$

SUMMARY

You can **write a system of linear inequalities from a graph** by identifying the equation for its boundary line and determining the inequality symbol needed. The symbols "less than" and "less than or equal to" indicate a shaded region below the line. The symbols "greater than" and "greater than or equal to" indicate a shaded region above the line. The symbols "greater than or equal to" and "less than or equal to" indicate a solid boundary line. The symbols "greater than" and "less than" indicate a dotted boundary line. When **writing a system from a scenario**, it is important to define each variable and determine any restrictions.